Reply by Author to F.R. Goldschmied

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THE Cebeci-Smith boundary-layer method is a numerical method which is found to give results which are satisfactory for most engineering problems. There is nothing special about it. The numerical scheme is an accurate and economical one, and the algebraic turbulence model has been correlated with available experimental data to produce reasonable results. This method is not expected to handle both the separation location and the resulting pressure distribution at separation, since such a capability will require an iteration between an inviscid and a viscous flow. What I mentioned in my paper was our observation of calculating the separation point by this method for a *given pressure distribution.* I am sure Mr. Goldschmied will agree with me, that flow separation changes the pressure distribution. So, to what extent the given inviscid pressure distribution for a flow with separation can be used is questionable. On the other hand, in using an inviscid pressure distribution I feel that one can locate the separation point for both laminar and turbulent flows by using zero-c_f criterion. Furthermore, I find it ridiculous to accept that the $c_f=0$ criterion for separation depends on chordwise location only. Actually, neither the chordwise location nor the pressure drop should be used directly to predict separation. Separation is the result of boundary-layer development and occurs when the wall shear goes to zero. Chordwise location and pressure drop at separation are results and not prerequisites for predicting separation.

Mr. Goldschmied claims that he and his associates have developed a method for predicting pressure distributions with massive separation. However, upon close examination, I find their method to be another self-predicting empirical correlation. Because the mystic Goldschmied separation criterion fails to predict the true separation pressure level, he adds a correction which also changes the separation location. So far, so good, but then he proceeds to test his method on cases which formed the basis for his empirical correlation! No independent test cases were presented. What else can one expect but good agreement? This is a good example of a circular argument.

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Comments on the Notion of "Loft Ceiling"

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Nomenclature

a = speed of sound
C = highest usable lift coefficient ($C_{L_{max}}$)
E = specific energy $E=h+V^2/2g$
g = acceleration of gravity
h = altitude
h_L = loft-ceiling altitude
M = Mach number
p = static pressure
q = dynamic pressure defined in Eq. (2)
V = aircraft velocity
W/S = wing loading
γ = ratio of specific heats
λ_1 = density scale height in exponential atmosphere
λ_2 = pressure scale height in exponential atmosphere
ρ = air density
σ = density ratio

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IN the series of stimulating papers on differential turning games[1-3] H.J. Kelly has introduced the notion of "loft-ceiling" as one of the parameters defining the capture sets of the game. The loft-ceiling h_L is defined as the highest altitude permitting vertical equilibrium in level flight. Neglecting the secondary thrust effect, the vertical equilibrium is expressed by

$$qSC \geq W \tag{1}$$

where q, the dynamic pressure, can be given by two equivalent expressions

$$q = \tfrac{1}{2}\rho V^2 = (\gamma/2)pM^2 \tag{2}$$

The loft-ceiling altitude is defined either by the corresponding density $\rho(h_L)$

$$\rho(h_L) = 2(W/S)/V^2C \tag{3a}$$

or by the static pressure $p(h_L)$ at that altitude

$$p(h_L) = 2(W/S)/\gamma M^2C \tag{3b}$$

The second expression is more adequate for cases where the maximum lift coefficient is Mach number dependent.

In French textbooks a similar notion called "plafond de sustentation" (or "lift-ceiling") is frequently used. It has been pointed out[4] that for aircraft of subsonic design the value of $M^2C(M)$ is bounded, indicating the existence of an absolute limit. For supersonic aircraft $M^2C(M)$ may have a maximum at transonic speed, but even though it recovers for higher Mach numbers its value is monotonically increasing. For such aircraft there is no absolute ceiling for vertical equilibrium; the ceiling is merely state dependent.

In the differential turning games[1-3] a reduced order aircraft model is used in which the relevant state variable is the specific energy $E=h+V^2/2g$. The "loft-ceiling" has to be interpreted therefore as the limiting altitude for vertical equilibrium, depending on the aircraft's specific energy $h_L(E) \geq h$.

Substituting in Eq. (1) the specific energy into the dynamic pressure q leads to a direct explicit relationship between the specific energy and the loft-ceiling altitude

$$E = h_L + \frac{gC}{W/S}\frac{1}{\rho(h_L)} = E(h_L) \tag{4}$$

This relationship is defined by the intersection of the line of a given specific energy with the line of C which designates, for a given configuration, the boundary of 1g engine-off stalling speed. The slope of the curve $E(h_L)$ is given by

$$\frac{dE}{dh_L} = 1 - \frac{W/S}{gC}\frac{1}{\rho}\left(\frac{1}{\rho}\frac{d\rho}{dh_L} + \frac{1}{C}\frac{dC}{dh_L}\right) \tag{5}$$

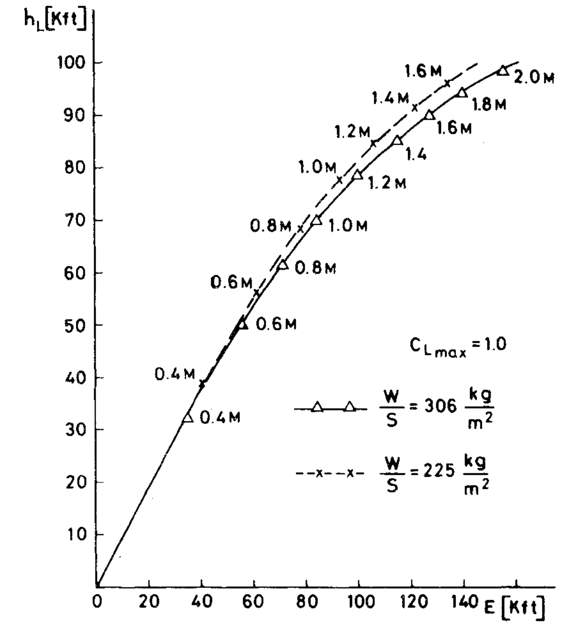

Fig. 1 Loft ceiling vs specific energy, different wing loadings.

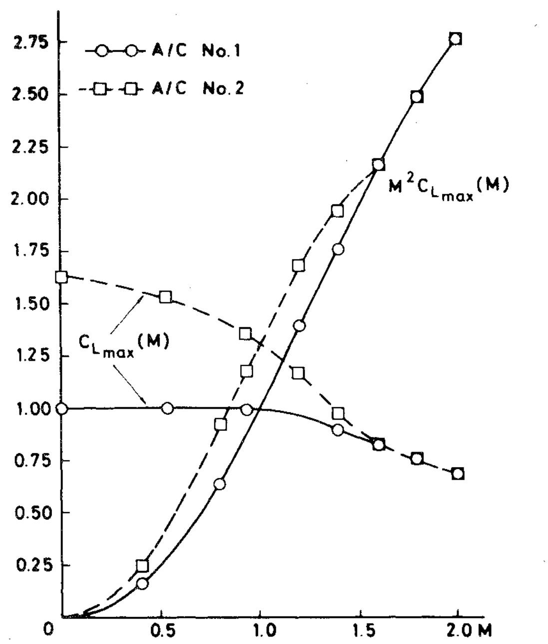

Fig. 2 Maximum usable lift coefficient vs Mach number.

Assuming subsonic stalling speed (constant C) and exponential atmosphere, the slope yields

$$\frac{dE}{dh_L}=1+\left(\frac{W/S}{\rho_0 C}\right)\frac{1}{g\lambda_1}\cdot\frac{1}{\sigma(h_L)} \tag{6}$$

The expression in the parenthesis is the half square of the equivalent airspeed for power-off 1g stall. This is the only aircraft configuration-dependent parameter, and varies in the narrow range of 120-160 knots for most known fighters. Moreover, the magnitude of the whole second term in Eq. (6) is of the order of 2-4% at sea level. Therefore, contrary to the statement of Kelley,[1] the close proximity of the curves of loft-ceiling vs specific energy of two entirely different aircraft at low energy is *not* coincidental.

The two curves tend to separate at higher altitudes. Here the stalling speeds become transonic, giving significance to eventual differences in aerodynamic design represented by

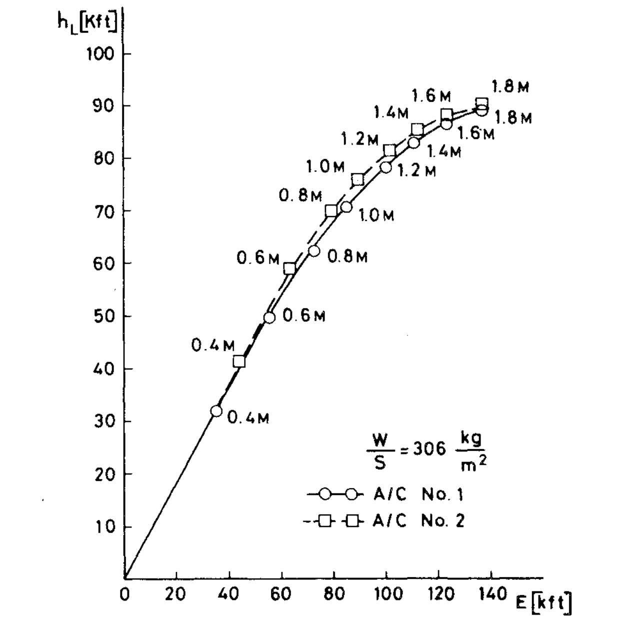

Fig. 3 Loft ceiling vs specific energy, different aerodynamic designs.

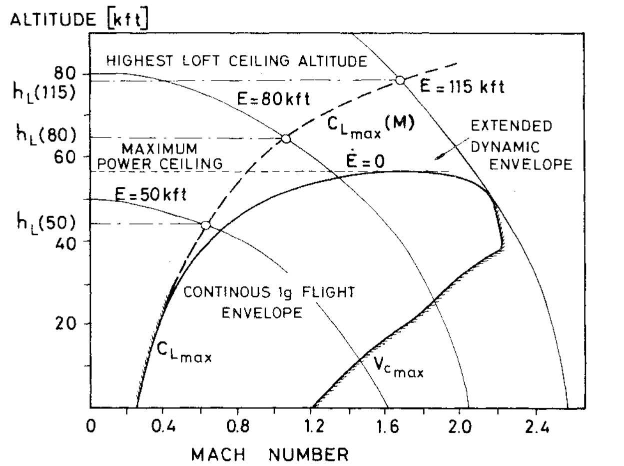

Fig. 4 Geometric definition of the loft ceiling.

$C(M)$. For the case where the maximum lift coefficient is Mach dependent

$$\frac{dE}{dh_L}=1+\frac{W/S}{gC(M)}\frac{1}{\rho(h_L)}\left[\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\frac{2(W/S)}{\gamma C_L(M)}\frac{1}{p(h_L)}\times\left\{\frac{dC(M)}{dM}\Big/\frac{d}{dM}[M^2C(M)]\right\}\right] \tag{7}$$

This last equation is the consequence of differentiating Eq. (3b) and assuming an exponential pressure altitude model with a scale-height λ_2. It can be slightly simplified by introducing M_{st} the Mach number of 1g power-off stall

$$\frac{dE}{dh_L}=1+\frac{\alpha^2(h_L)}{2g\lambda_1}M^2_{st}\left[1-\frac{\lambda_1}{\lambda_2}M^2_{st}\times\left\{\frac{dC(M)}{dM}\Big/\frac{d}{dM}[M^2C(M)]\right\}\right] \tag{8}$$

The second term in the parenthesis may have a magnitude of 0.1-0.2, depending on the particular aerodynamic design.

Two demonstrative examples are given to show the influence of various aircraft design factors on the $h_L(E)$ relationship. First, two aircraft of different wing loadings are compared. Both are assumed to have maximum lift coef-

Table 1 Aircraft characteristics

A/C No.	W/S, kg/m^2	C_L	V_{st}, m/sec
1	225	1.0	60
2	306	1.0	70

ficients not influenced by Mach number resulting in different values of stalling speeds (see Table 1).

The loft-ceiling characteristics of these two aircraft are depicted in Fig. 1, showing only an almost negligible difference at low energy levels which increase slightly with altitude. The Mach numbers at loft-ceiling altitude are also marked indicating that transonic speed is reached only at high altitudes.

The second example demonstrates the effect of Mach dependent maximum lift coefficient on the loft-ceiling. Aircraft with the same wing loading but with different maximum lift characteristics (see Fig. 2) were chosen, indicating the improvements due to modern aerodynamic design (strakes, leading-edge flaps, etc.).

The loft-ceiling characteristics of these two substantially different aircrafts in Fig. 3 show however only very slight variations.

Concluding Remarks

The loft-ceiling concept is of importance to show the possible dynamic extension of the flight envelope in aerial combat (see Fig. 4) and it is used as a criterion to determine capture in air combat modeling.[1-3] However, it seems that most fighter aircraft have similar loft-ceiling characteristics. This observation indicates that requirements for loft-ceiling advantage at the same specific energy level may be difficult to implement.

References

[1]Kelley, H.J., "Differential-Turning Optimality Criteria," *Journal of Aircraft*, Vol. 12, Jan. 1975, pp. 41-44.

[2]Kelley, H.J., "Differential-Turning Tactics," AIAA Paper 74-815, Mechanics and Control of Flight Conference, Anaheim, Calif., Aug. 1974.

[3]Kelley, H.J., "Differential-Turning Maneuvering," 6th IFAC Conference, Boston, Mass., Aug. 1975.

[4]George, L., Vernet, J.F., and Wanner, J.C., *La Mechanique du Vol*, Dunod, Paris, 1969.

Reply by Author to Shinar, Levin, and Marari

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THE results reported in the preceding technical comment are of interest, and it is indeed true that comparisons between fighter aircraft characteristics at equal loft ceilings tend to be generally much the same as equal-energy comparisons, at least when the aircraft are conventional, i.e., neither has thrust-augmentation-of-lift features. Loft ceiling appears in "energy" approximation treatment of aircraft flight dynamics as a state-dependent control bound, viz. the upper bound on altitude, a control variable, dependent upon the state, specific energy. It represents the highest altitude that can be sustained in a short-term sense, i.e., ignoring any energy loss that may result from insufficient thrust to maintain energy, with the roller-coaster altitude-velocity interchange dynamics omitted from the model (which omission is the crux of energy approximation).

It is interesting that in a more sweeping approximation, a different ceiling quantity appears (see the section of Ref. 1 subtitled "Tactics with Models Further Reduced in Order"). It might be called "composite ceiling" and consists of either the loft ceiling or the power ceiling (determined as the horizontal-equilibrium bound), whichever is lower at the energy under examination. For typical fighters, it is comprised of the loft ceiling up to a certain energy, the power ceiling at higher energies. There is, of course, a certain artificiality in the distinction between loft ceiling and power ceiling whenever the power ceiling is higher, i.e., one should not take simplified drag models very seriously at high angles of attack, as lift saturation is usually accompanied by drag build-up much faster than quadratic.

Reference

[1]Kelley, H.J., "Differential-Turning Tactics," *Journal of Aircraft*, Vol. 12, Dec. 1975, pp. 930-935.

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